Purpose: In this problem set, you will review the skills of rational functions we have built so far for the purpose of being able to graph rational functions.

Let's start with some review.

1. Consider the rational function below.

$$f(x) = \frac{(x^2 - 64)(x + 2)(x - 4)}{(x^2 - 16)(x + 2)(x + 8)}$$

   (a) Find the domain of $f(x)$ in interval notation.

   (b) Find the vertical asymptotes of $f(x)$.

   (c) Using a calculator, fill out the table below. What is the behavior of the vertical asymptote at $x = -4$?

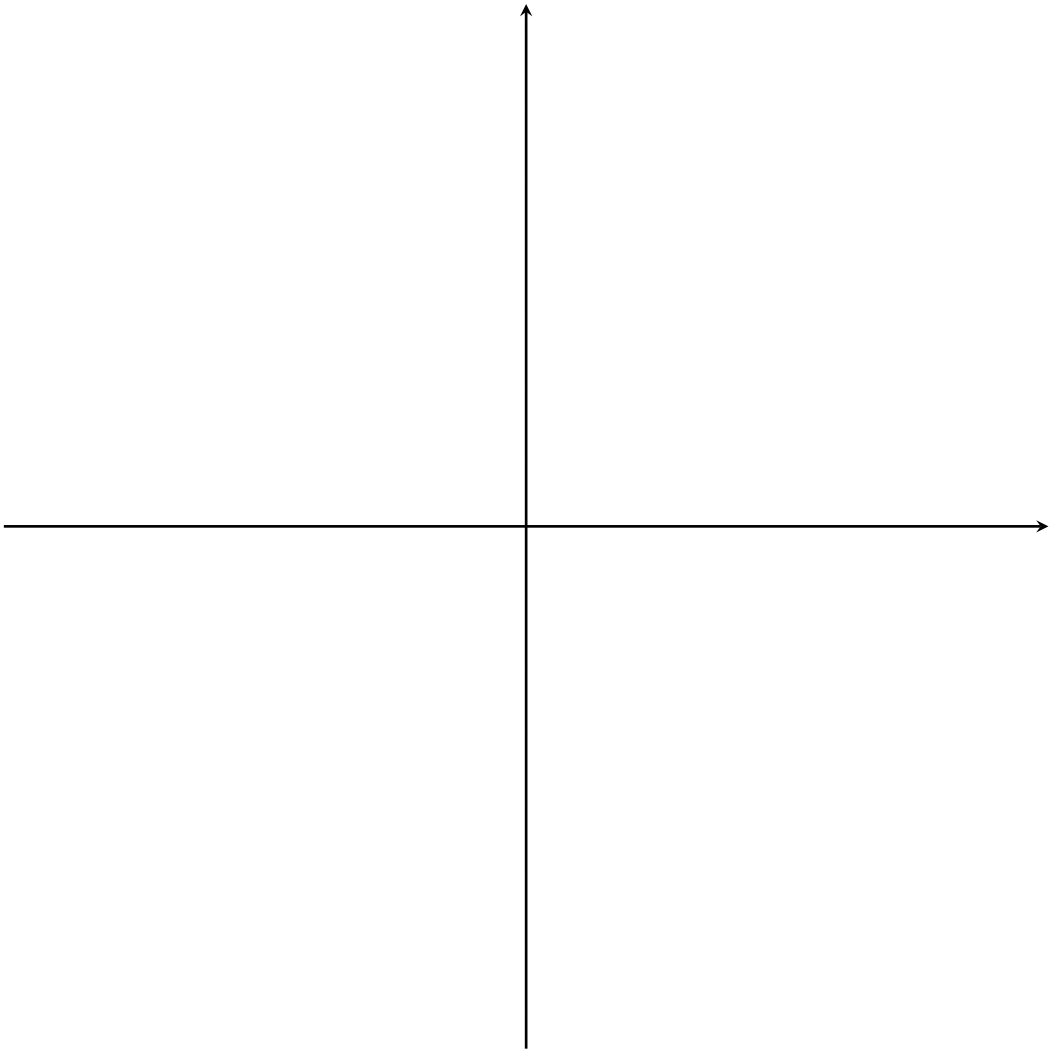
   (d) Does $f(x)$ have any holes? Where?

   (e) What is the end behavior of $f(x)$?

   (f) What are the x-intercepts of $f(x)$? What is the y-intercept of $f(x)$?

We have *almost* everything we need to sketch a graph of $f(x)$! The last thing we need are some points on each connected piece of the graph (which we often call **connected components**).

(g) On the graph below, piece everything together that you found above and use the blank space to find some points on each component of the graph. (Feel free to use a calculator.) Make sure you label at least two points on each piece of the graph, indicate asymptotes and holes, and label horizontal and vertical intercepts.



That's it! That's how we graph these functions. We sometimes refer to this method as "DVAXYHAPP" which corresponds to

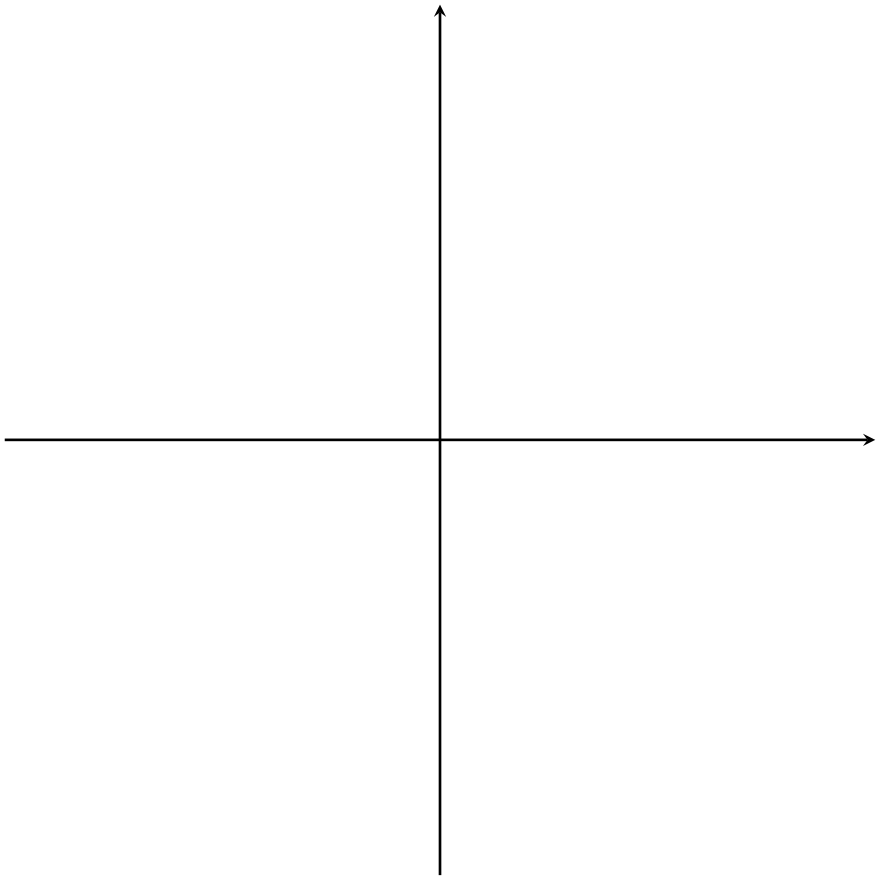
- **D**omain
- **V**ertical **A**symptotes
- **X** and **Y** intercepts
- **H**orizontal **A**symptotes
- **P**lot **P**oints

Below is a list of functions. Graph each of them on the pages that follow using the **DVAXYHAPP** method.

- $f(x) = \dfrac{(x+2)(x-3)(x+4)}{(x^2-x-6)(x-9)}$
- $g(x) = \dfrac{(x-4)(x+6)}{(x^2-9)^2(x-1)}$
- $h(x) = \dfrac{(x^2-36)(x-4)}{(x+6)}$
- $k(x) = \dfrac{(x+2)(x-4)}{x+1}$
- $\ell(x) = \dfrac{(x+3)(x-2)^2(x-1)}{x(x-3)^2}$

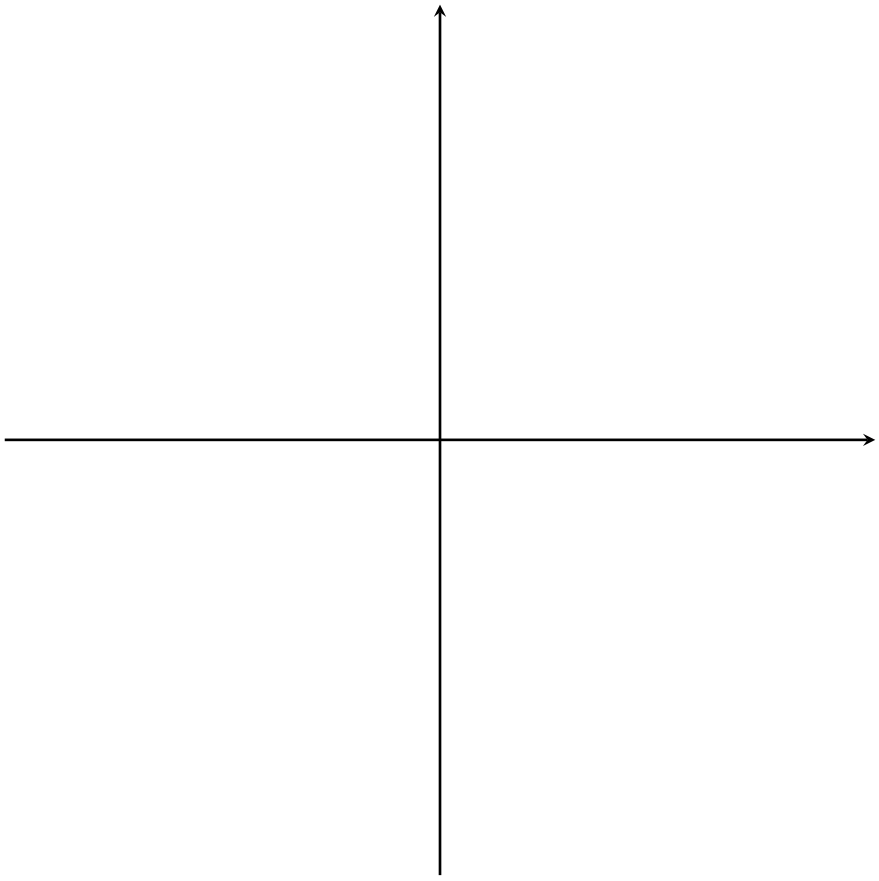
Function we're sketching:

Organized work to sketch:



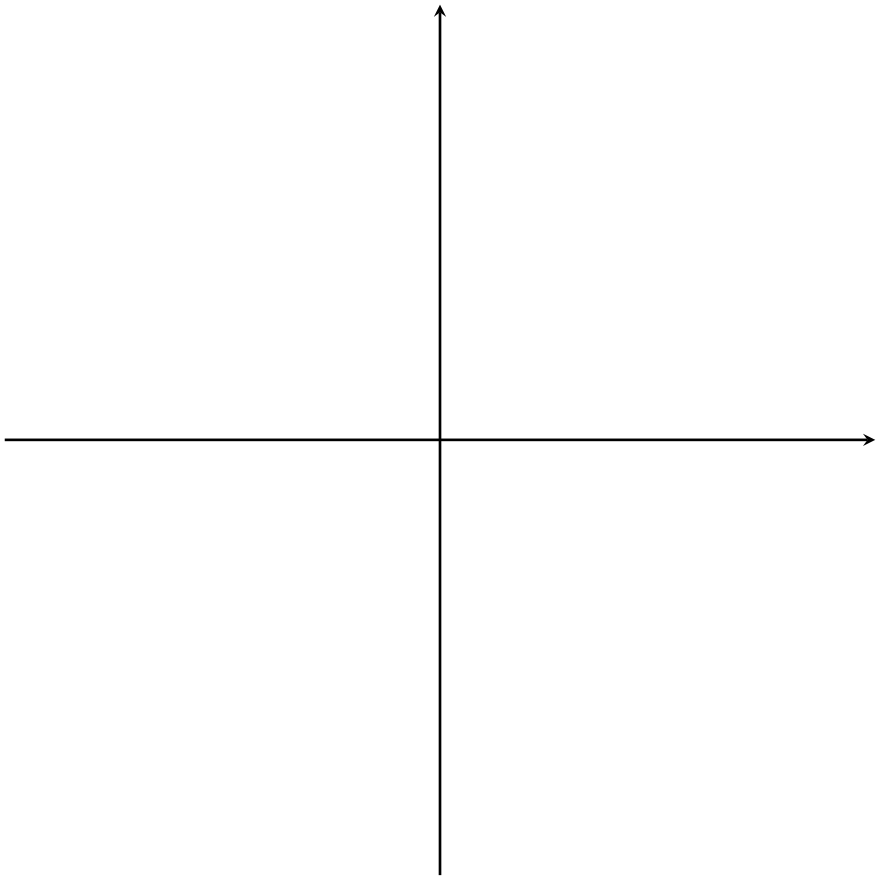
Function we're sketching:

Organized work to sketch:



Function we're sketching:

Organized work to sketch:



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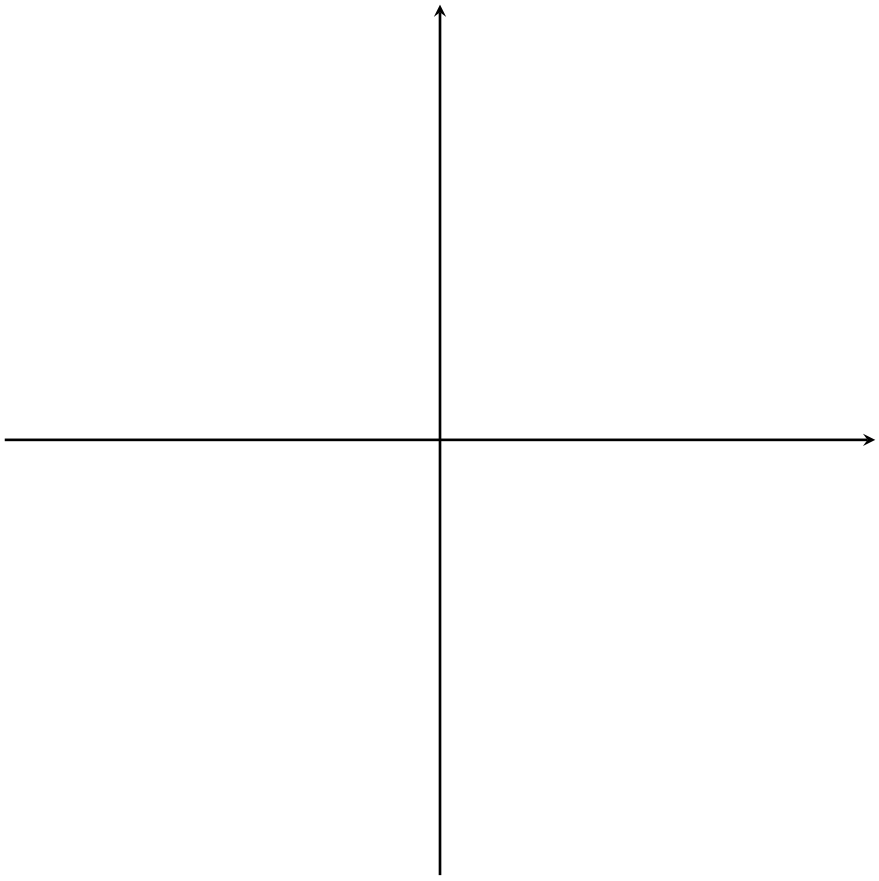

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